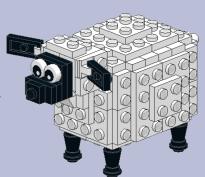
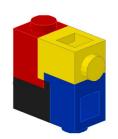
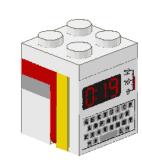
Brick Geometry



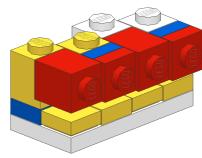


BrickCon 2020 Online





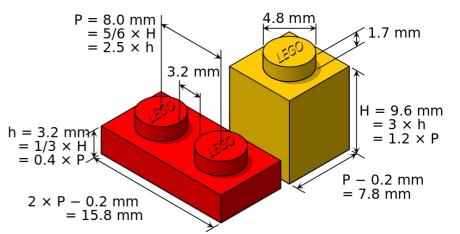
bill@wards.net www.brickpile.com





© 2008-2020 William R. Ward Some Rights Reserved License: http://creativecommons.org/licenses/by-nc-sa/3.0/us/

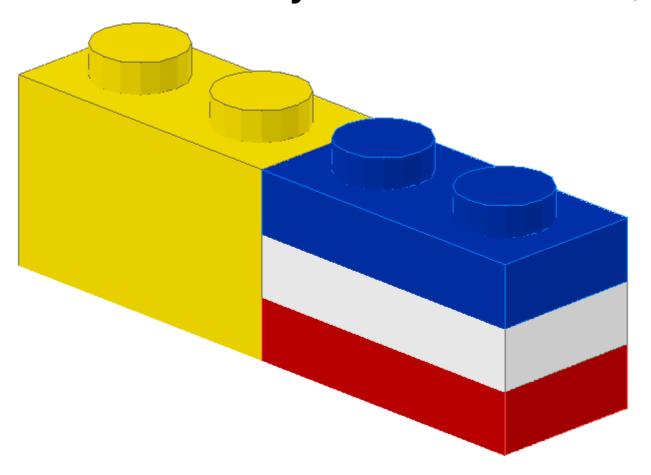




Ratios and Dimensions

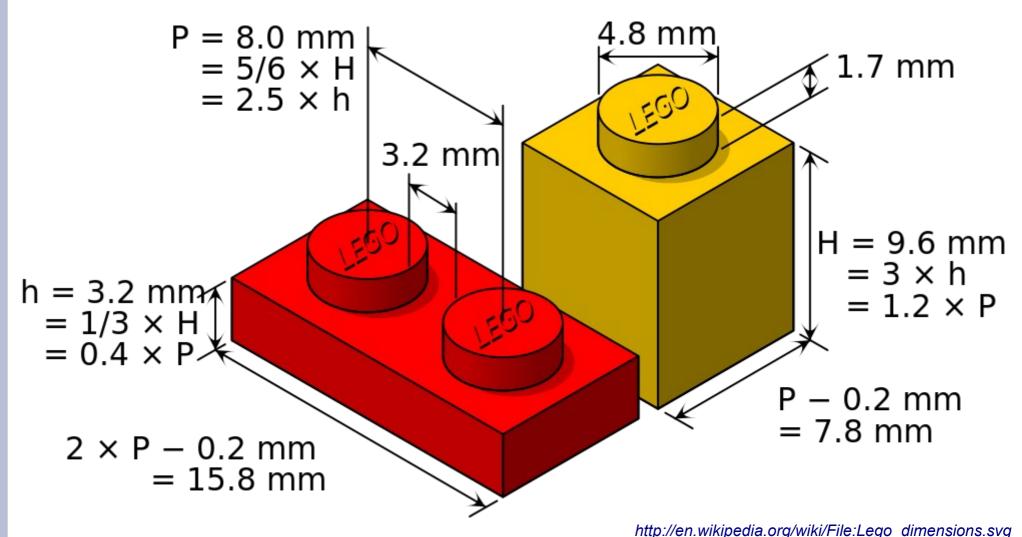
Basic LEGO Geometry 1 brick = 3 plates

Everyone knows this, I hope...



LEGO Dimensions

Quick! Memorize all these numbers. There will be a quiz later.



Math is Hard! LDU = LDraw Unit

- A 1x1 stud brick or plate is 5/16" or 8mm (0.8cm)
- The height is 6/16" or 9.6mm (0.96cm)
- To make the math easier, LDraw designers came of up with the LDraw Unit (LDU)
- Everything is a nice, easy integer this way!

	<u>LDU</u>	<u>studs</u>	<u>bricks</u>	<u>plates</u>	<u>cm</u>	inch	<u>pt</u>	20
LDU	1	1/20	1/24	1/8	0.04	1/64	9/8	
studs	20	1	5/6	5/2	8.0	5/16	45/2	
bricks	24	6/5	1	3	0.96	6/16	27	
plates	8	2/5	1/3	1	0.32	2/16	9	
cm	25	1.25	1.04	3.125	1	0.39	28.3	
inch	64	3.2	8/3	8	2.54	1	72	
pt	8/9	2/45	1/27	1/9	0.0353	1/72	1	

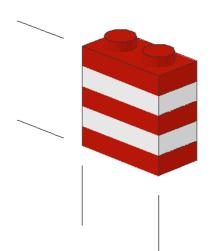
LEGO Bricks Are Not Square

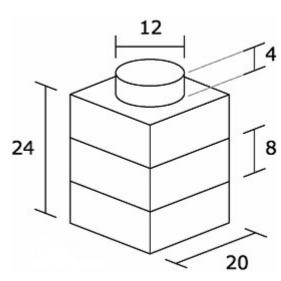
Bricks are 8mm wide by 9.6mm high How do you make widths and heights match?

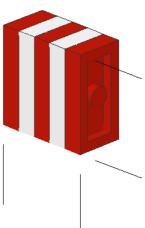
LDU makes the math easy....

How many plates = how many studs?

- 2 studs = 2x20 = 40 LDU
- 5 plates = 5x8 = 40 LDU



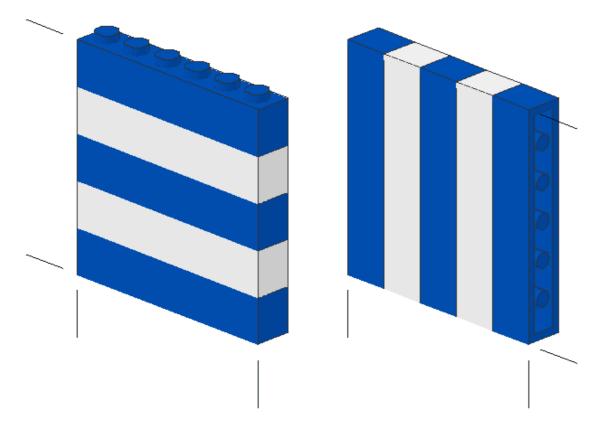


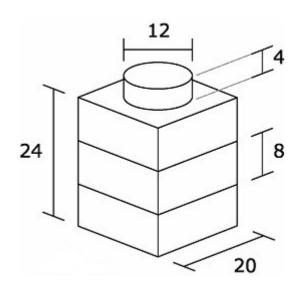


6:5 Brick Ratio

How many bricks = how many studs?

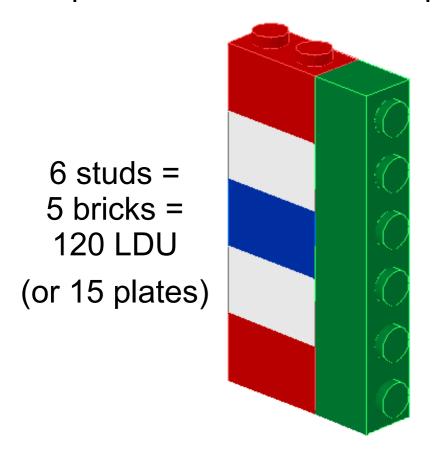
- 6 studs = 6x20 = 120 LDU
- 5 bricks = 5x24 = 120 LDU

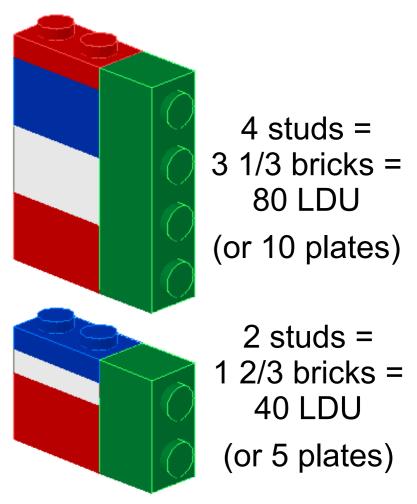




Even Numbers of Studs

Any even number of studs corresponds to a combination of bricks and plates, since 2 studs = 5 plates

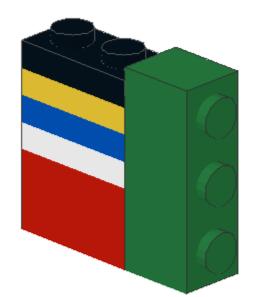




Odd Numbers of Studs

Since 1 stud = $2\frac{1}{2}$ plates, no combination of plates adds up to

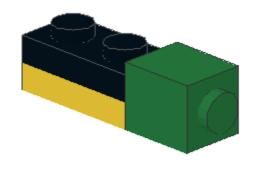
exactly one stud



3 studs = 60 LDU

1 brick + 4 plates = 56 LDU

(Missing ½ plate!)

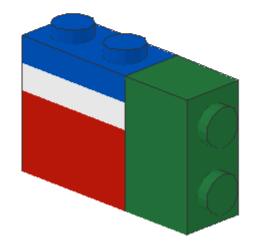


1 stud = 20 LDU

2 plates = 16 LDU

?? = 4 LDU

(Missing ½ plate!)



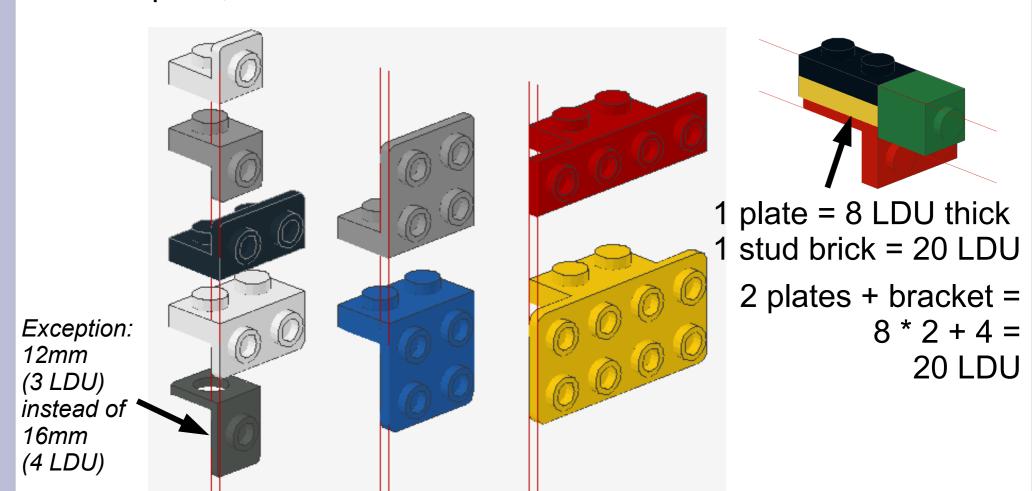
2 stud = 40 LDU

5 plates = 40 LDU

(OK)

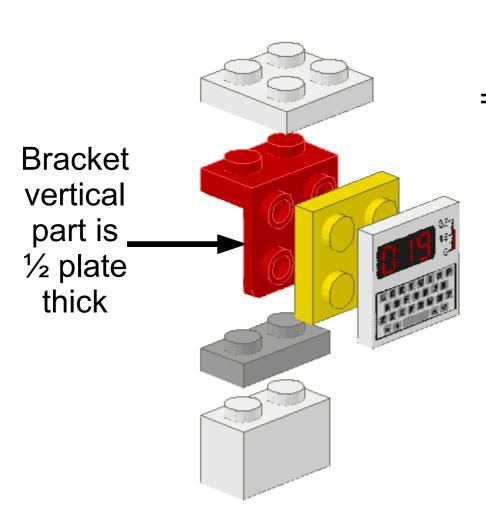
But where do you get ½ plate?

One answer: brackets. The thin vertical plate is ½ the thickness of a normal plate, or 4 LDU

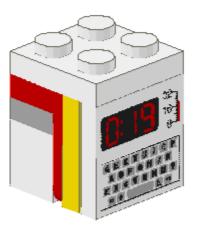


SNOT Studs Not On Top

Using ½ plate thickness from brackets



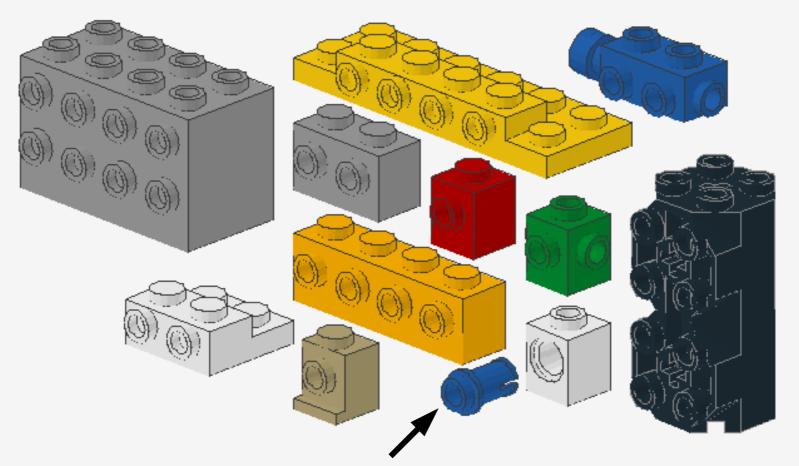
½ plate from bracket
+ 1 plate + 1 tile
= 2 ½ plates, same as 1 stud



Computer tile is flush with edge of white 2x2 plate.

Bricks with Studs on the Side

LEGO has made a number of parts over the years with studs on the side(s), useful for SNOT (Studs Not On Top) design.



* Technic brick with ½ pin is not quite the same – see next page

Caveat - Technic Pin Alignment



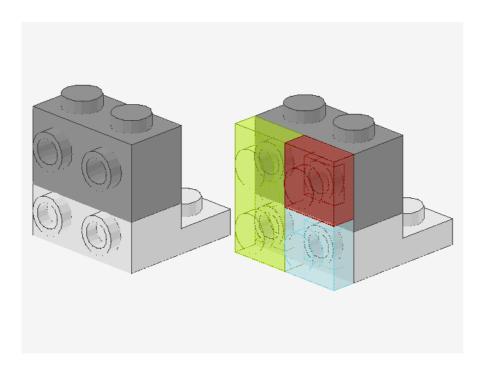
Technic pin hole placement is just a little higher (about 0.2mm) than stud-on-side placement.

Some models may have alignment problems due to this.

Reason: early Technic brick molds needed thicker plastic between pinhole and bottom of brick.

Result: LEGO now produces more bricks with studs on side instead of using ½ pins in Technic bricks – better for us anyway

2+3=5



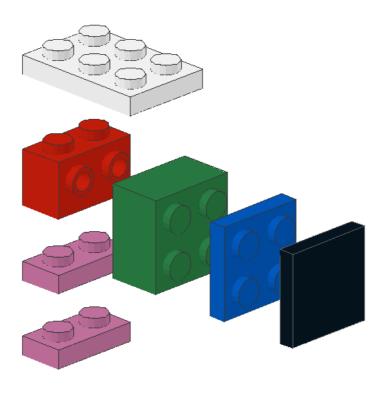
White "Plate, Modified 2 x 2 x 2/3 with 2 Studs on Side" (part 99206) = 2 plates high

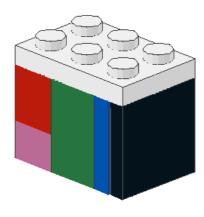
Grey "Brick, Modified 1 x 2 with Studs on 1 Side" (part 11211) = 3 plates high

Total = 5 plates high = 2 studs wide

Bricks with studs on sides to mount flush

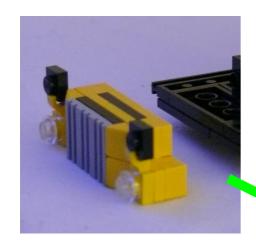
Use bricks with studs on sides to attach assemblies at 90 degrees. To mount them flush, remember that 5 plates = 2 studs = 40 LDU.





Flush tile examples

My Lunar School Bus model used this technique.



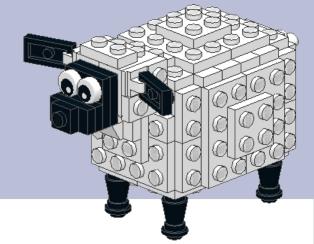
Grille and headlight sub-assembly fits flush in 2-stud space

5 plates

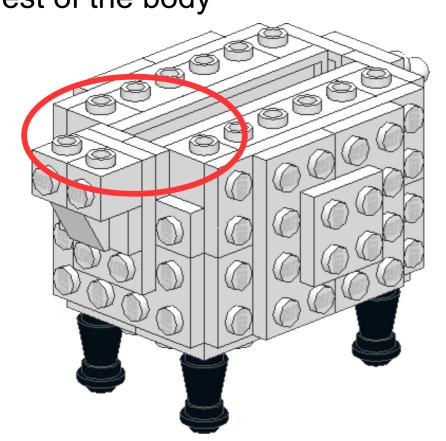
- = 40 LDU
- = 2 studs

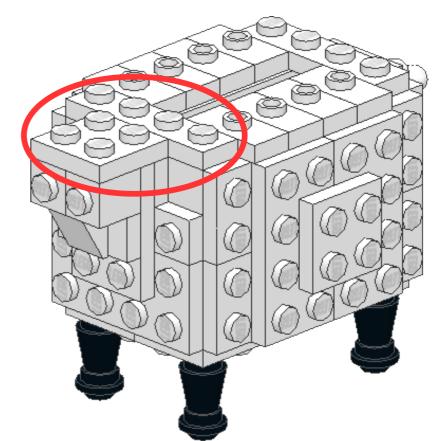


Another Alignment Example

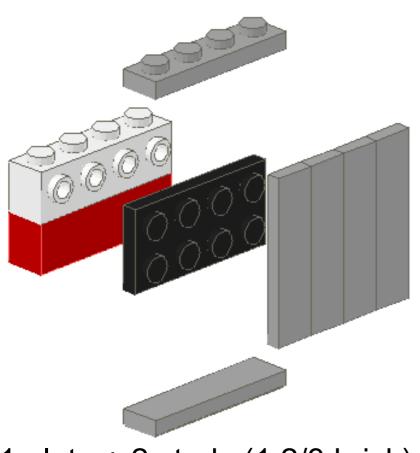


Note how the neck is made from a brick and two plates = 40 LDU So we can place a 2x2 plate on top and the studs line up with the rest of the body

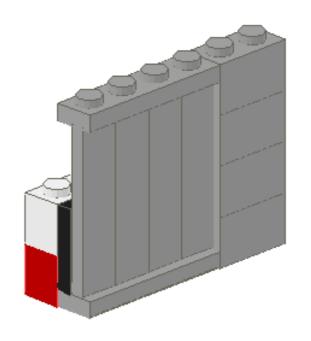




Inset Panels



1 plate + 2 studs (1 2/3 brick) = 2 bricks Use half-plate offsets to add texture to an otherwise flat wall



Tiles are ½ plate inset

2 plates + 4 studs (3 1/3 bricks) = 4 bricks

Inset panels example



Panels inset by ½ plate

My F40PH Caltrain locomotive

Photo by Drew Dirschell: https://www.flickr.com/photos/7333042@N06/4639456044

"De Vier Gekroonden"

This model by Vincent "Mr. Tomato Bread" Kessels uses some of these techniques.

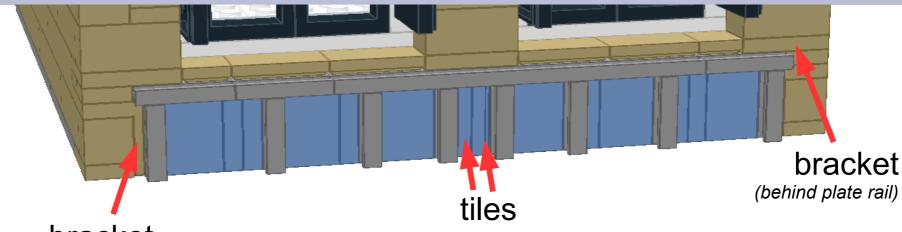
Photos used with permission

Model:
"De Vier Gekroonden"
by Vincent Kessels
a.k.a.
"Mr. Tomato Bread"





Sideways Building with Brackets



bracket

Stack bricks and plates with a tile on the end Mount on bracket on each side facing inward

Model: 7 n/a

"De Vier Gekroonden" by Vincent Kessels

"Mr. Tomato Bread"

 $3 \text{ bricks} = 3 \times 24 = 72 \text{ LDU}$

7 plates + 1 tile = $8 \times 8 = 64 \text{ LDU}$

bracket = 4 LDU

Total = 140 LDU = 7 studs

Tiles on the ends press together and friction holds it all together

The Problem with Jumper Plates

Requirement: 5-stud wide window openings with 6-stud wide arches above.

Solution: offset the arches by ½ stud using jumper plates

Side effect: How to fill ½ stud gap on ends?

Imperfect solution: attach tile mounted sideways.

½ stud = 10 LDU. Tile = 8 LDU. Gap of 2 LDU (¼ plate) cannot be filled by any LEGO part. Any better ideas to fill a 10 LDU space?

Model:

"De Vier Gekroonden" by Vincent Kessels a.k.a.

"Mr. Tomato Bread"



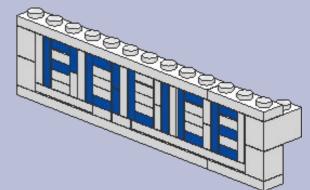
Mosaic Dates on LEGO Modular Sets

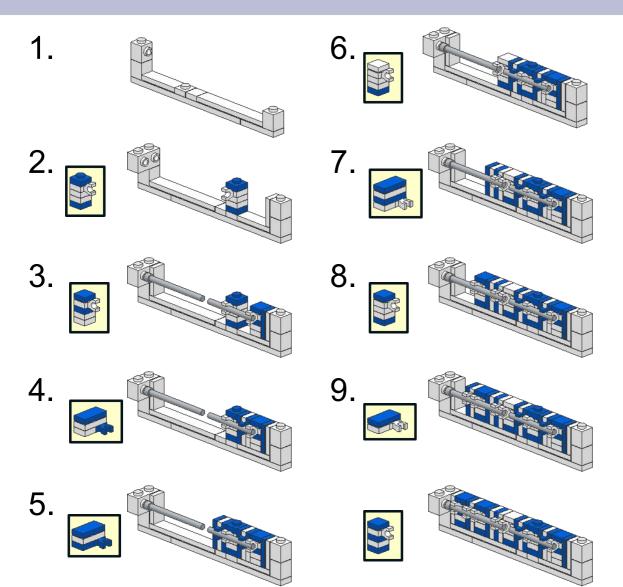
These sets (#10197 & 10224) use plates and tiles to create SNOT dates on the buildings.





Clip and Bar Lettering





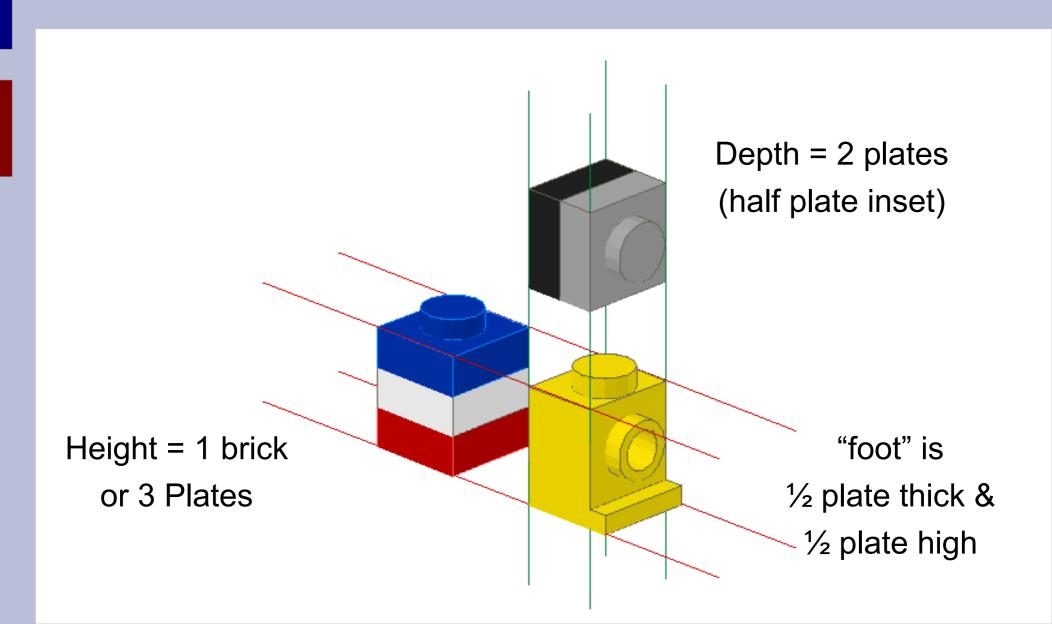
Example of lettering style similar to that from the fire house and town hall sets.

Using this clip and bar technique frees you from worrying about spacing.

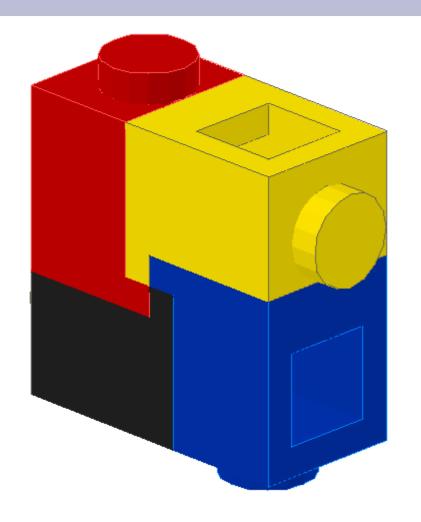
Only the middle of the "O" lines up with the studs above and below!

Fun with Headlight Bricks

Headlight Brick Dimensions



Four headlight bricks

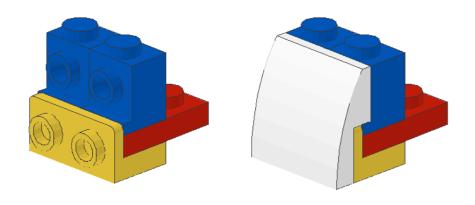


2 plates (red) +
3 plates (yellow) =
5 plates = 2 studs

Result: 5 plates or 2 studs in each of 4 directions.

Combining Brackets & Headlight Bricks

1/2 plate offset on headlight bricks + 1/2 plate thickness on bracket = 1 plate thickness, suitable for 2x2 curved slope

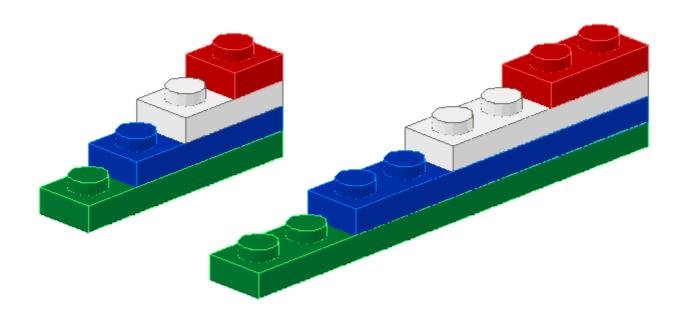




As seen in the LEGO IDEAS Ghostbusters set #21108 (minifig display)

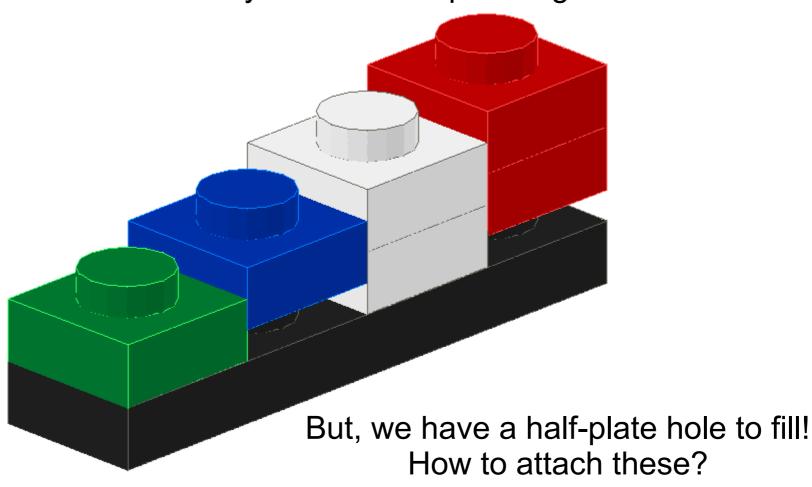
Problem: Gradual Steps

How do you make a gentle slope? What if these are too steep?



Gradual Steps

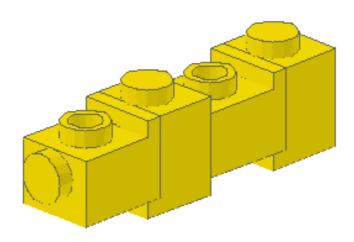
For a more gradual slope, we'd like to mount every other one ½ plate higher

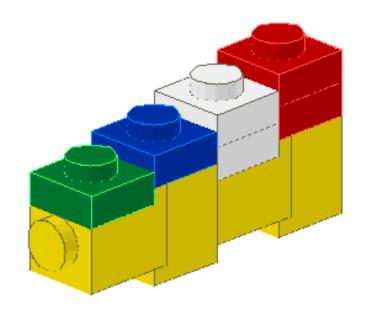


Solution: Headlight Bricks

Alternate rotations for headlight bricks to take advantage of ½ plate offset in "foot"

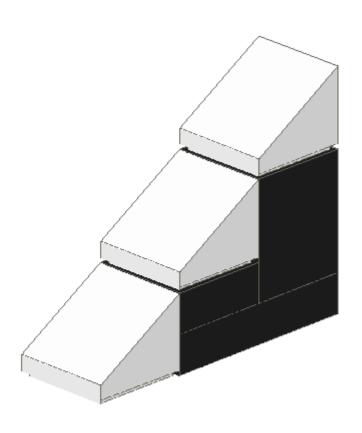
2 plates + $\frac{1}{2}$ plate = 1 stud





Half-plate lift from "foot"

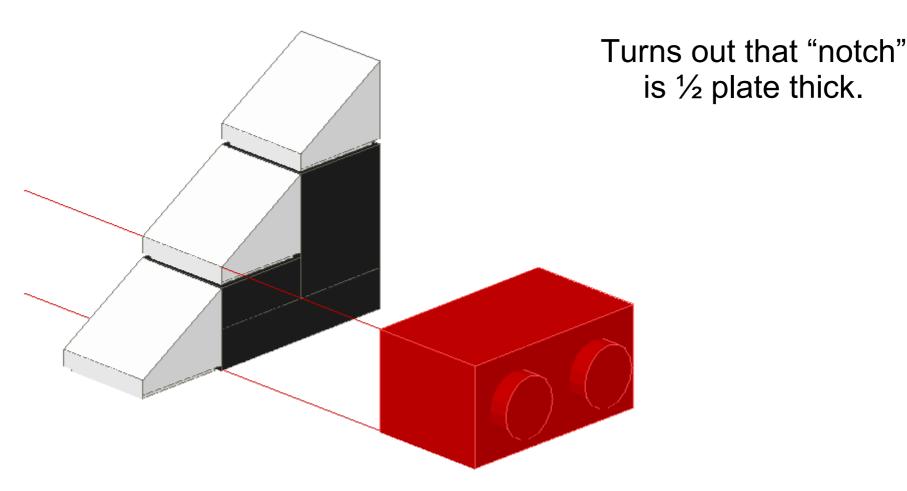
Problem with "cheese slope": Stairstep effect



The 1x1 "cheese slope" is a very useful part but doesn't combine well with others of its kind to make a smooth slope.

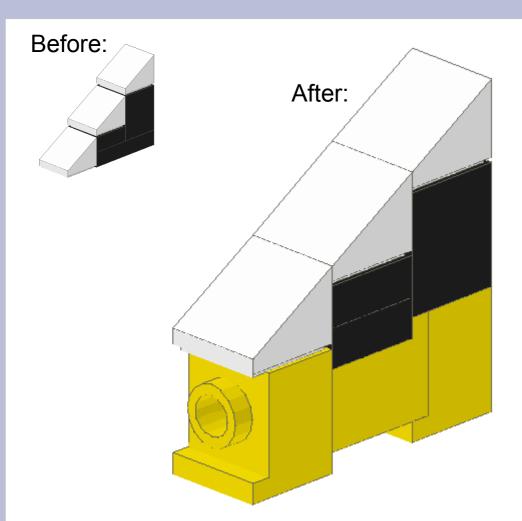
This notch is needed for it to fit a stud inside, but is ugly.

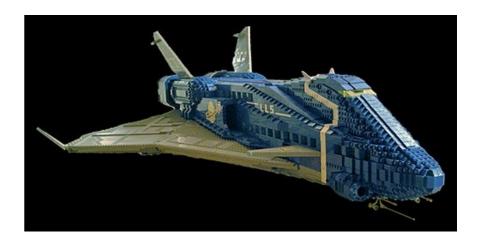
Problem with "cheese slope": Stairstep effect



2 plates (height of cheese slope) + ½ plate = 1 stud

Solving the stairstep effect





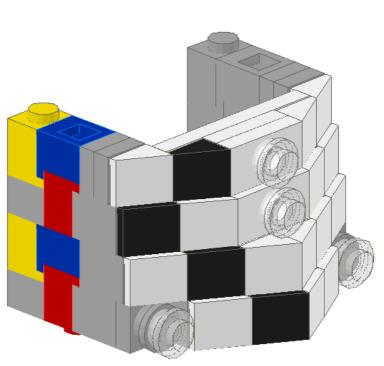
Used in Bram Lambrecht's "Legoland Spacelines 979"

> http://www.flickr.com/photos/bram/1461137007/ (used with permission)

Mount the center slope ½ plate lower for a smooth surface!

Useful for trains, too







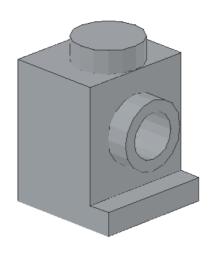
My F40PH Caltrain locomotive

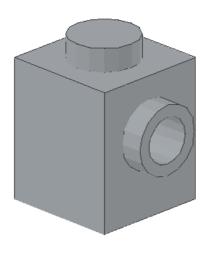
"Headlight Brick" vs. "Brick 1 x 1 with Stud on 1 Side"

Headlight Brick depth = 2 plates = 16 LDU

Brick depth = 2 ½ plates = 20 LDU

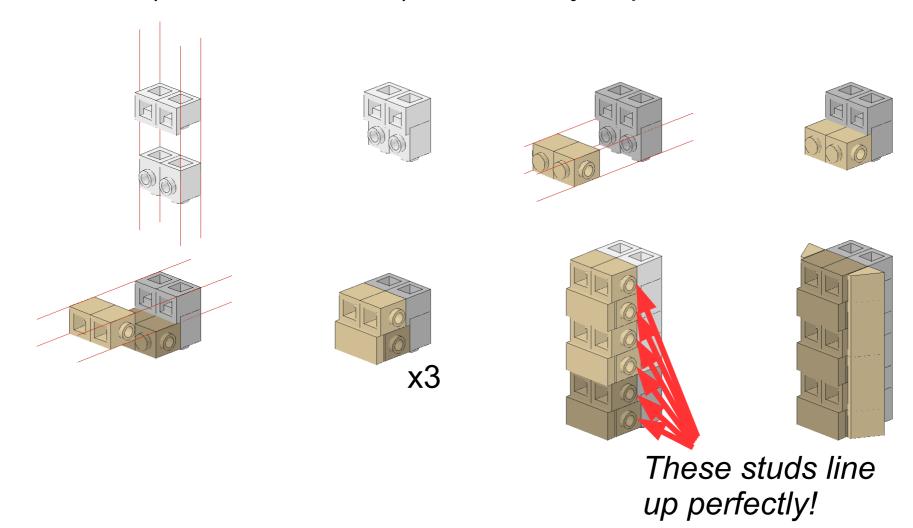
Combine these to achieve ½ plate differences in depth!



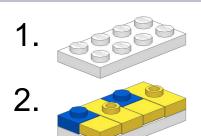


Hospital Bay Window example

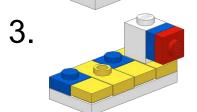
Windows (bottoms of bricks) are inset by ½ plate



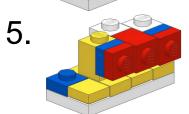
How about QUARTER plate offsets?



Remember from Vincent Kessels' building that a jumper plate adds a 1/4 plate offset?



4.



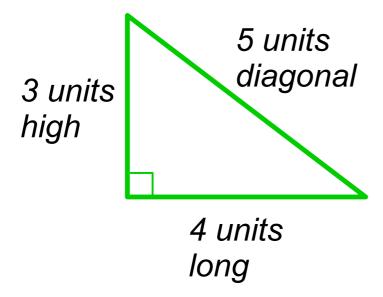
6.

Combine that with the ½ plate difference between the "headlight brick" and the "brick with stud on one side" and get a *very* gradual step.

Triangles

Pythagorean Triples

Pythagorean Formula
$$a^2 + b^2 = c^2$$



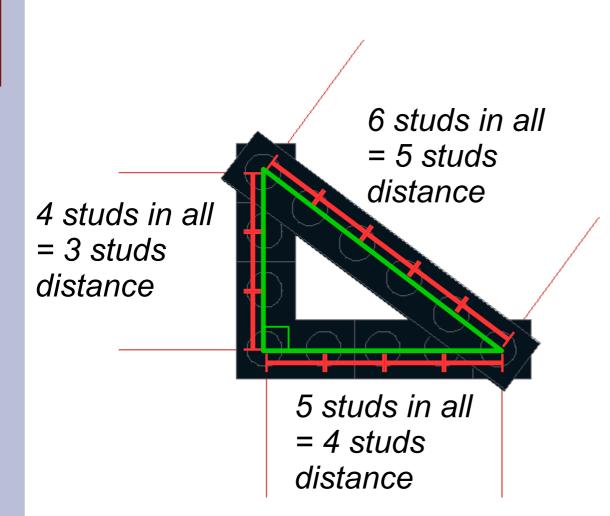
Pythagorean Triples are right triangles where the sides are all integers.

The first Pythagorean
Triple is the 3-4-5 triangle.
As long as the 3 sides
have this ratio it will have a
perfect right angle.

Multiples also work: 6-8-10, 9-12-15, etc.

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

Pythagorean Triples in LEGO

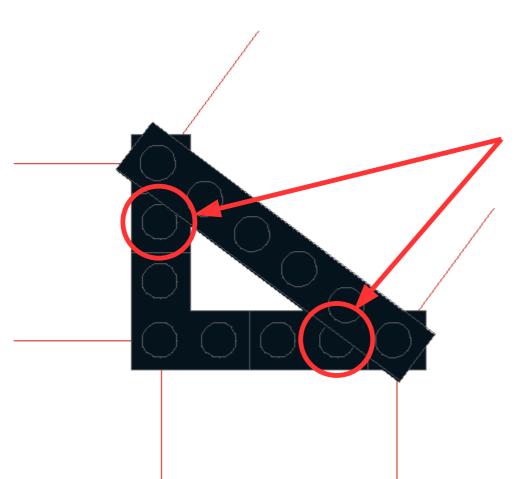


When making a Pythagorean Triple in LEGO, add one stud to each side!

Why? Count between the centers of the studs!

$$(3, 4, 5)$$
 triangle = $(4, 5, 6)$ studs

Pythagorean Triples in LEGO



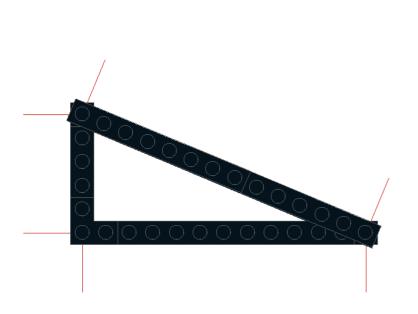
Note: See how the studs are partly blocked by the diagonal.

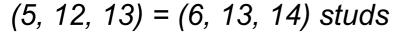
This triangle must be at least three plates thick, due to the spacer plates needed between the diagonal and the horizontal or vertical plates.

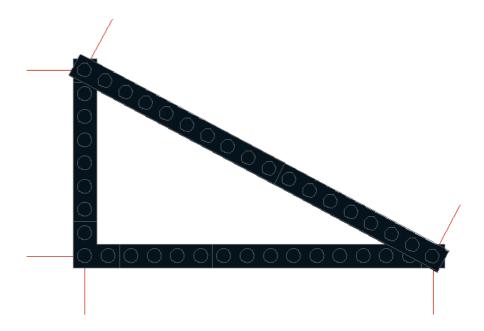
Or use Technic beams, 1x4 plate with only 2 studs, etc.

More Pythagorean Triples

- There are only 4 triples of diagonal length 25 or less:
 - (3, 4, 5); (5, 12, 13); (8, 15, 17); (7, 24, 25)
- Any other triangle with integer sides will not be a right triangle!
- LEGO Examples:



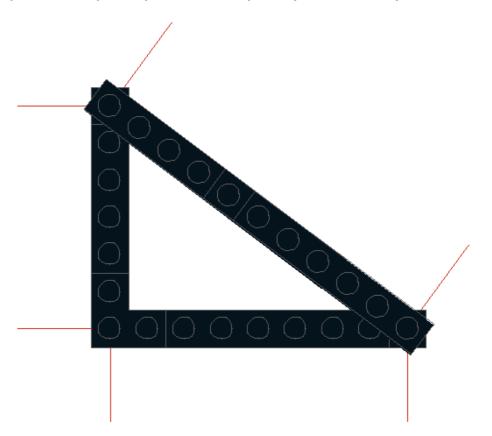




(8, 15, 17) = (9, 16, 18) studs

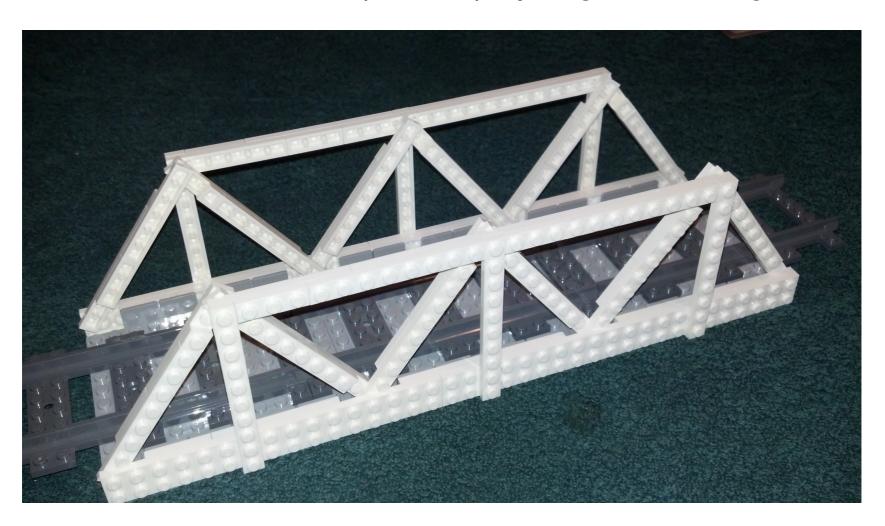
Multiples of Pythagorean Triples

- Unit does not have to be 1 stud! Additional ones can be made by multiplying these values by a scaling factor.
 - Example: 2x(3, 4, 5) = (6, 8, 10) = (7, 9, 11) studs



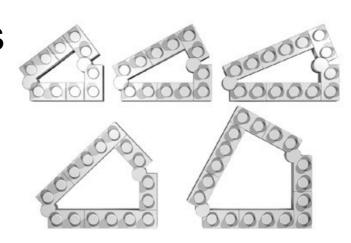
Pythagorean Triple Example: Truss Bridge

Trusses made from (6, 8, 10) Pythagorean triangles



Triangles from Hinges

Another way to achieve triangles is to use a hinge element. You can achieve many more possible angles using this technique.



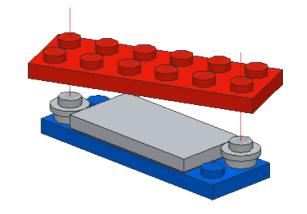
For more information including a list of possible angles, see: http://www.l3go.bugge.com/articles/technique/Hypotech1.shtml

Swap Corners Technique

Two identical rectangles have identical diagonal length. You can therefore place one atop the other, rotated so the corners touch.

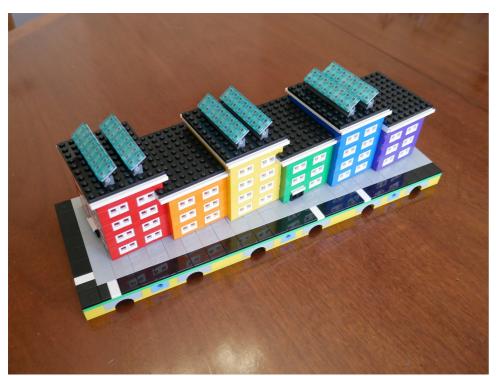


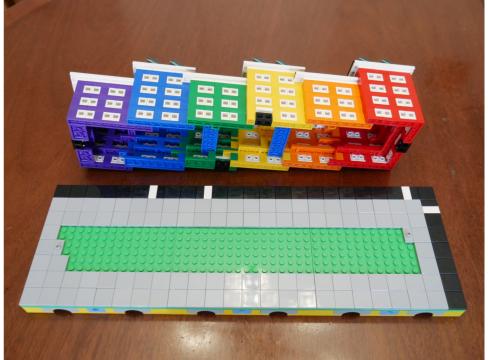
In LEGO, you can do this by connecting the corner studs similarly:



Swap Corners Example

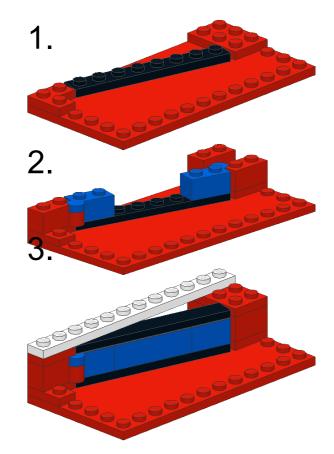
You can even mount an entire building this way...

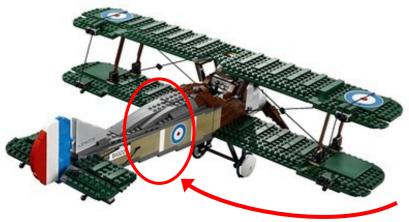




Swap Corners using Hinges

A related technique can be done using hinges, since the rotation point of the hinge is exactly on the corner of the brick.





Example: tapered rear of fuselage on LEGO aircraft such as Sopwith Camel (set 10226)



Studs in All Directions

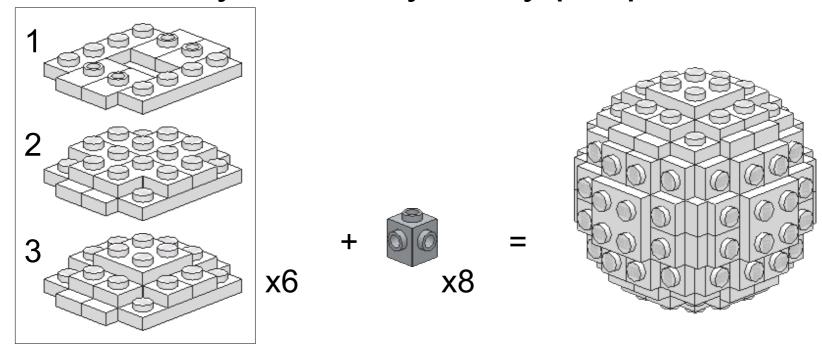
Travis Brick

- a.k.a. "Brick, Modified 1 x 1 with Studs on 4 Sides"
- Named the "Travis Brick" by the LEGO fan community after the late Space builder Travis Kunce, who had it tattooed on his arm
- Many "Studs on All Sides" techniques use this piece



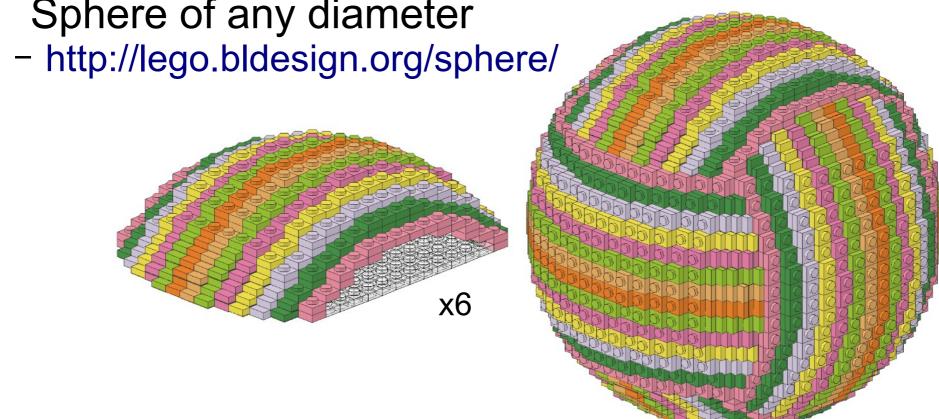
Lowell Sphere

- How do you build a sphere out of LEGO?
- Solution designed by Bruce Lowell for a 6.8stud diameter sphere (4 studs + 6 plates)
- Basis for many MOCs by many people



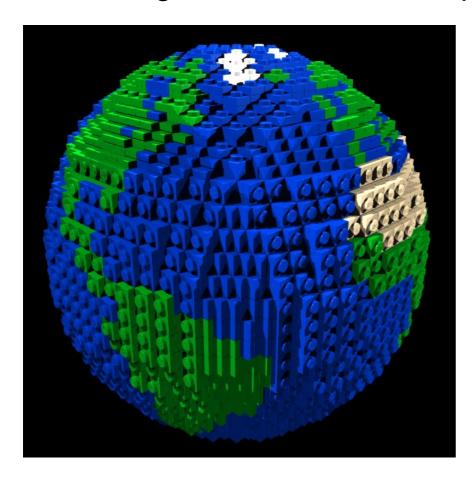
Bram's Sphere Generator

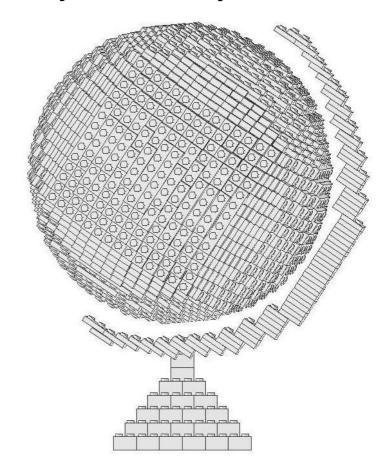
 Bram Lambrecht wrote a program to generate LDraw instructions for a Lowell Sphere of any diameter



LEGO IDEAS project: Globe

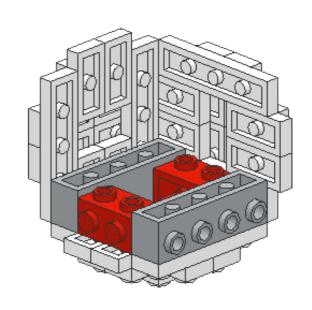
https://ideas.lego.com/projects/16205 (got 699 votes in 2012)
Globe design based on Lowell Sphere by "WWWally" from 2012





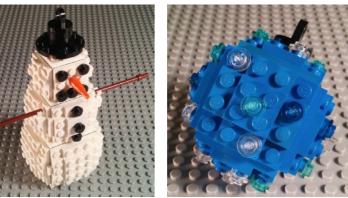
Travis Brick Not Required

- Note: For the 6.8 stud diameter Lowell Sphere and up, you can use other SNOT parts for the connection instead of the Travis Bricks.
- Tip: you can center the jumper plates on the hollow studs if using old style jumper plates.

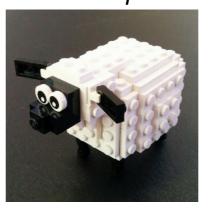


Some of my Lowell Sphere based MOCs

Christmas Ornaments



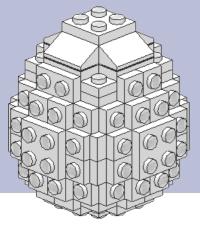
Sheep



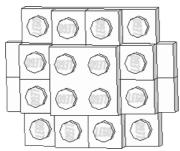


Kermit's eyes

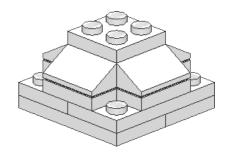




Breaking Eggs Going from Sphere to Oval

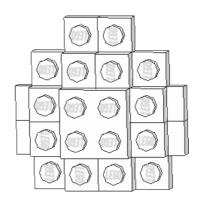


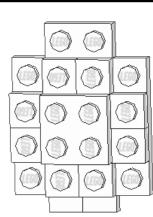
The round end (bottom) of egg is the same as on a standard Travis
Sphere



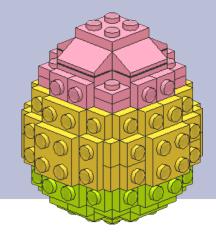
The pointy end (top) of the egg is my own custom design

The sides (2 of each version) are the same as on a standard Travis Sphere but with one edge extended

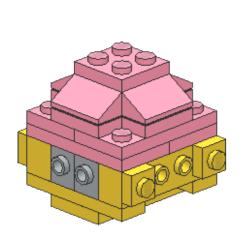


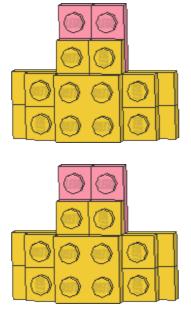


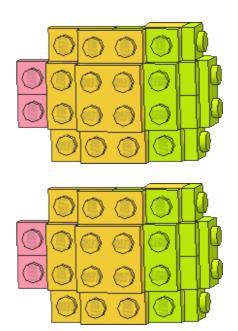
Coloring Eggs

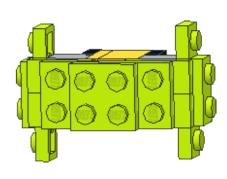


This egg has the *exact same shape* as the white one, but to get the colors to work, and to work around the limited range of parts in pink, the structure is very different!









Taking it to the Next Level

Parts of the Easter Bunny, Stay-Puft, and Teddy Bear are built in a similar way, just not a spherical shape...





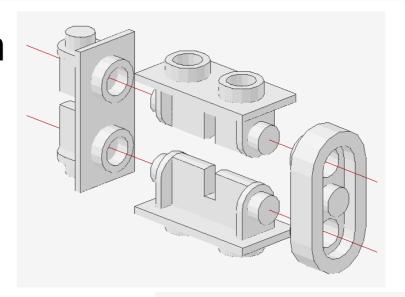


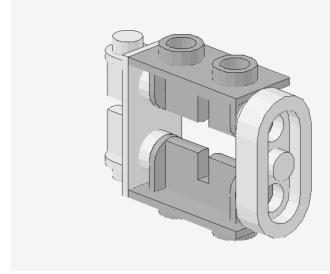
Miscellaneous Brick Techniques

DSnot

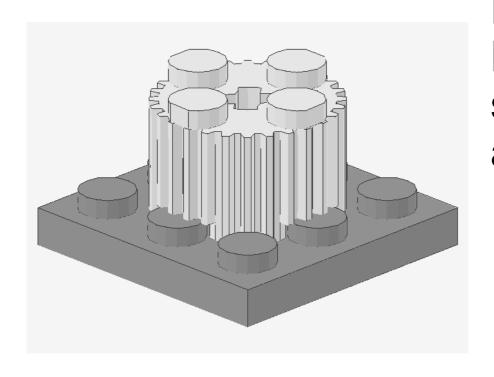
"DSnot" technique from swooshable.com shows a fun use of the 1x2 hinge

- Can use any hollow stud parts on ends. I replaced one hinge with a "Plate, Modified 1 x 2 Rounded with 2 Open Studs" (Part 35480)
- Hinges can still be rotated!





Surprise 45° Connection



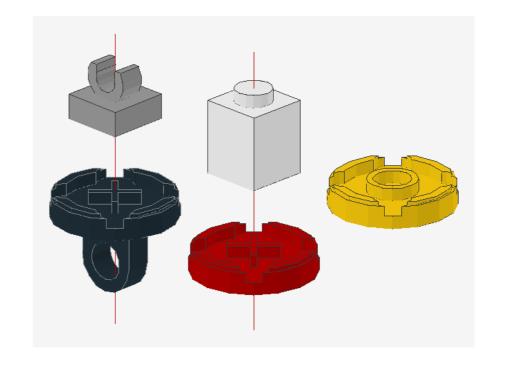
Brick, Round 2 x 2 with Flutes (Grille) and Axle Hole (92947) fits on studs at a 45 degree angle!

- Due to the "flutes" the cutouts in the corners on the bottom are just the right size
- Does not work with smooth round parts

Another Weird & Illegal 2x2 Round Part Connection

The old style 2x2 round tile with "cross" (and the version with Lifting Ring) can attach to a 1x1 brick

- Can connect to any 1x1 square part
- They changed it though the modern 2x2 round tile has a circle on the underside instead.



Additional Resources

- Reinhard Beneke, BrickFest PDX '04 http://www.brickshelf.com/cgi-bin/gallery.cgi?f=74539
- Sir Bugge's Hypo-Techniques (traingles from hinges): http://www.l3go.bugge.com/articles/technique/Hypotech1.shtml
- The New Elementary, blog about new parts https://www.newelementary.com/
- Swooshable, website about clever building techniques https://swooshable.com/
- Bruce Lowell's "Lowell Sphere" page http://www.brucelowell.com/lowell-sphere/
- Bram Lambrecht's sphere generator http://lego.bldesign.org/sphere/
- Slides from this talk (and previous versions), from my website: http://www.brickpile.com/tag/brick-geometry/

Q & A

Thank you

Contact me if you have any further questions...

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(c) @billwardsbrickpile